Approximation of ECG Signals using Chebyshev Polynomials

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Abstract: The ECG (Electrocardiogram) signal represents electrical activity of heart and is recorded for monitoring and diagnostic purpose. These signals are corrupted by artifacts during acquisition and transmission predominantly by high frequency noise due to power line interference, electrode movements, etc. Addition of these noise change the amplitude and shape of the ECG signal which affect accurate analysis and hence need to be removed for better clinical evaluation. In this paper, ECG signal taken from MIT -BIH database is first denoised using Total Variation Denoising (TVD); using Majorization minimization (MM) optimization technique. ECG signals generate massive volume of digital data, so they need to be suitably compressed for efficient transmission and storage. Hence, for efficient compression the signal is segmented into various sections using Bottom-Up approach. The individual sections are then approximated using Chebyshev polynomials of suitable orders. The performance of the approximation technique is measured by computing the Maximum Absolute Error, the Compression Ratio, Root Mean Square Error, Percent Root Mean Square Difference and Percent Root Mean Square Difference Normalized. The results are also compared with other techniques as reported in the literature, where significant improvements in all the performance metrics are observed.

Keywords: ECG signal, total variation denoising, majorization-minorization, bottom-up, Chebyshev nodes, Chebyshev approximation

1. Introduction

The electrocardiogram (ECG) describes the electrical activity of the heart. It conveys information about structure of the heart and functions of its electrical conduction (Walraven, 2011). It is obtained as voltage variations by placing electrodes at specific positions on the chest, arms and legs. The ECG is characterized by a series of waves whose morphology and timing provide clinically useful information. The time pattern that characterizes the occurrence of successive heartbeats is also very important. The ECG signals are used to monitor drug and to detect metabolic disturbances. A systematic interpretation of the ECG signals prevents overlooking of important abnormalities of cardiac system like rhythm of heartbeats, size and position of chambers and the presence of any damage to the heart's muscle cells or conduction system, the effects of cardiac drugs, and the function of implanted pacemakers (Braunwald, 1997).

A single normal cycle of the ECG represents the successive atria depolarization/repolarisation and ventricular depolarization /repolarisation which occur with every heartbeat (Acharya et. al, 2007). These can be approximately associated with the peaks and troughs of the ECG waveform labelled P, Q, R, S, and T as shown in Figure 1.

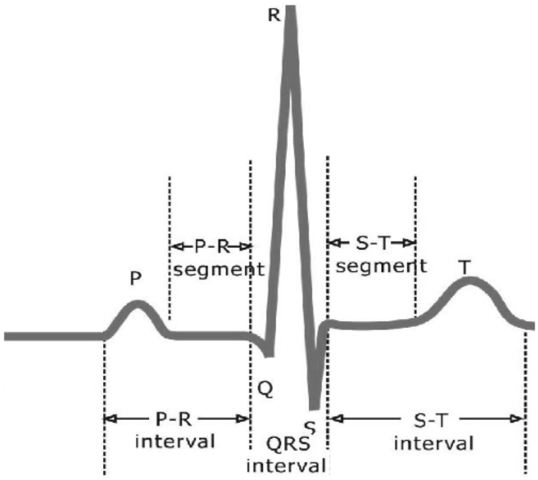

Figure 1. ECG Signal and its various waves.

The first step in the design of an ECG system involves understanding the nature of the signal that needs to be acquired. The ECG signal consists of low amplitude voltages in the presence of high offsets and noise. The common electrode used in ECG systems has a maximum offset voltage of 300mV. The actual desired signal is 0.5mV superimposed on the electrode offset. ECG Signal processing is a huge challenge since the actual signal value will be 0.5mV in an offset environment of 300mV. Other factors like AC power-supply interference, RF interference, electrode movement from surgery equipment, and implanted devices like pace makers and physiological monitoring systems can also impact accuracy (Bharadwaj and Kamath, 2011).

The amount of data involved in storage and transmission of digital ECG signals is quite large. So it needs to be adequately compressed in a way so that it can be accurately reconstructed. The ECG compression techniques are broadly classified as: direct methods, transform-based methods and parameter extraction methods (Jalaleddine et. al, 1990). In direct methods, the original ECG signal samples are compressed directly, and in transformation method the original samples are first transformed and then compressed. In parameter extraction methods, the features of the processed signal are extracted and then these features are used for the reconstruction of the signal (Jalaleddine et. al, 1990).

Various time domain compression algorithms like FAN, AZTEC, CORTES and SAPA etc can be found in the literature. These methods are based on the idea of extracting few significant signal samples to represent the signal and then decoding the same set of samples. These techniques are based on heuristics in the sample selection process. This makes them faster but suffer from suboptimality (Zahhad, 2011).

Several compression algorithms including polynomial approximations and polynomial interpolation have been proposed for ECG data compression. The advantage of polynomial approximation is that it requires only polynomial coefficient describing the data signal and is able to approximate the original ECG signal quite efficiently.

Polynomials of maximum degree 3, including spline functions have been proposed for ECG interpolation in (Karczewicz and Gabbouj, 1997). The representation of ECG signals using second degree quadratic polynomials is studied by Nygaard et al in (Nygaard et. al, 1999). High degree Legendre polynomials were used for ECG data compression (Philips, 1993; Colomer and Colomer, 1997; Tchiotsop et. al, 2007). Although Chebychev polynomials are widely used in mathematical interpolation and approximation, ECG signal compression through Chebychev polynomials are hardly found in the literature. In (Tchiotsop et. al, 2007) ECG data compression is done using Discrete Chebyshev Transform by segmenting the signal into blocks which consist of multiple cardiac cycles.

These methods are mainly focused on approximation of the entire ECG beat without paying attention to the importance of the intervals of the signal which is the case for vital signals. The signal is often broken into segments within which the signals can be considered stationary. In this way, each part can be analyzed or processed separately.

In this paper, we propose a computationally efficient method to model ECG signals through Chebyshev polynomials. The ECG signal contains an important noise component so a preprocessing is applied before the segmentation effectively takes place. The ECG signal is first denoised using Total Variation Denoising using Minimization-Majorization (TVD-MM) technique. In order to have a better compression ratio we must have a lower order of the polynomial. So, the denoised signal is then segmented into segments using Bottom-Up approach. Next, we model each segment independently using Chebyshev interpolation and combine them to reconstruct the complete signal.

The rest part of the paper is organized as follows: in section 2, we present the computational performance metrics to be applied to measure the efficiency of the method. In section 3 and 4, we give a brief introduction to Chebychev polynomials and Chebyshev interpolation. In section 5, we describe the proposed method along with the algorithm to achieve ECG data compression. In section 6, we present the implementation of our method on the ECG signals using the MIT-BIH arrhythmia database and discuss the results obtained. In the last section we give the conclusions regarding the presented approach.

2. Evaluation of Compression Method

Let N be the total number of the ECG samples and $p(x_i)$ and $f(x_i)$ represent the samples in original and reconstructed signal. In most ECG compression algorithms, the various performance metrics are as:

Root Mean Square Error ($RMSE$): is the average of the square of the errors (Sormno and Laguna, 2006)

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(p(x_i) - f(x_i))^2}{N}} \tag{1}$$

Maximum Error ($Maxerror$): is the maximum error (Hadjileontiadis, 2006)

$$MaxError = \max_{N}(|(p(x_i) - f(x_i))|) \tag{2}$$

Compression Ratio (CR): is defined as the ratio between the number of bits needed to represent the original and the compressed signal (Zahhad et. al, 2010). We can also define it as the ratio between the number of samples needed to represent the original and the compressed signal.

$$CR = \frac{N}{n} \tag{3}$$

where n is the order of the interpolating polynomial

Percent Root Mean Square Difference (PRD): is the measure of acceptable fidelity (Zahhad et. al, 2010)

$$PRD\% = 100\sqrt{\frac{\sum_{i=1}^{N}(p(x_i) - f(x_i))^2}{\sum_{i=1}^{N} p(x_i)^2}} \tag{4}$$

Percent Root Mean Square Difference Normalized ($PRDN$): is the normalized version of PRD (Fira and Goras, 2008) and depends on the signal mean value p_m

$$PRDN\% = 100\sqrt{\frac{\sum_{i=1}^{N}(p(x_i) - f(x_i))^2}{\sum_{i=1}^{N}(p(x_i) - p_m)^2}} \tag{5}$$

3. Chebyshev polynomials

The Chebyshev polynomials (Gil et. al, 2007) of first type and degree n are defined in terms of trignometric cosine function as:

$$T_n(x) = \cos(n\cos^{-1}(x)) \text{ for } n \geq 0 \tag{6}$$

The expressions for Chebyshev polynomials are obtained from the recursive relation

$$\begin{aligned} T_0(x) &= 1 \\ T_1(x) &= x \\ T_{n+1}(x) &= 2xT_n(x) - T_{n-1}(x) \text{ for } n \geq 1 \end{aligned} \tag{7}$$

The Chebyshev polynomials of degree n, $T_n(x)$, has $n+1$ zeros in the interval [-1, 1], which can be calculated as

$$x_j = \cos\left(\frac{2j+1}{2n+1}\pi\right),\ 0 \leq j \leq n \tag{8}$$

The roots of the Chebyshev polynomial are also known as Chebyshev points or nodes. In the same interval the $n+1$ extrema of the polynomial $T_n(x)$ are located at

$$\widetilde{x_j} = \cos\left(\frac{\pi j}{n}\right),\ 0 \leq j \leq n \tag{9}$$

At all the maxima $T_n(x) = 1$, while at all the minima $T_n(x) = -1$. The Chebyshev polynomials are orthogonal in the interval $[-1, 1]$ over the weight $w(x) = \left(1 - x^2\right)^{-1/2}$. They also satisfy discrete orthogonality relationships. Other properties of Chebyshev polynomials can be found in (Szego, 1975).

4. Polynomial Approximation using Interpolation

A polynomial approximation problem is of finding a polynomial close to a given function and has the freedom to select the significant points. Once the significant points have been fixed, it is reduced to an interpolation problem that can be solved by polynomial interpolation (Birkhoff and Boor, 1965). Let $p(\mathbf{x})$ represent ECG segment vector of length N consisting of samples of $p(x_i)$ such that

$$p(\mathbf{x}) = \{p(x_0), p(x_1), ..., p(x_N)\}\,,\ \mathbf{x} \in [a, b]$$

Given a set of $N + 1$ data points $(x_i, p(x_i))$ we want to construct a polynomial f of degree N with the property

$$f(x_i) \approx p(x_i),\ i = 0, 1, ...N$$

Suppose the interpolation polynomial is in the form

$$f(x) = a_N x^N + a_{N-1} x^{N-1} + ... + a_2 x^2 + a_1 x + a_0,\ x \in [a, b] \tag{10}$$

which means that

$$f(x_i) \approx p(x_i) \forall i \in \{0, 1, ...N\} \tag{11}$$

Substituting Eq.(11) in Eq.(10) we get a system of linear equations which in matrix form reads

$$\begin{bmatrix} x_0^N & x_0^{N-1} & \cdots & x_0 & 1 \\ x_1^N & x_1^{N-1} & \cdots & x_1 & 1 \\ \vdots & \vdots & & \vdots & \vdots \\ x_N^N & x_N^{N-1} & \cdots & x_N & 1 \end{bmatrix} \begin{bmatrix} a_N \\ a_{N-1} \\ \vdots \\ a_0 \end{bmatrix} = \begin{bmatrix} p(x_0) \\ p(x_1) \\ \vdots \\ p(x_N) \end{bmatrix} \tag{12}$$

The matrix on the extreme left is the Vandermonde's matrix. The system given by Eq.(12) would have a unique solution if the determinant of the Vandermonde matrix does not vanish (Bjorck and Pereyra, 1970). Solving this system for a_k we can construct the interpolating polynomial $f(x)$.

Alternatively we can write the required polynomial explicitly using Lagrange's formula (Yang et. al , 2005; Chan et al, 2001) as

$$f(x) = \sum_{i=0}^{N} f(x_i) \prod_{\substack{j=0 \\ j \neq i}}^{N} \frac{x - x_j}{x_i - x_j}, x \in [a, b] \tag{13}$$

Let us now construct yet another interpolating polynomial $p(x)$ by sampling $f(x)$ at n interpolation points such that $n < N$. We can estimate the difference between them, i.e., the interpolation error $E(x)$. Let Π_n denote the space of polynomials of degree$\leq n$, and let $C^{n+1}[a,b]$ denote the space of functions that have $n+1$ continuous derivatives on the interval $[a,b]$. Then from the truncation error from the Taylor series, we have this theorem:

Theorem 1: Let $f(x) \in C^{n+1}[a,b]$. Let $p(x) \in \Pi_n$ such that it interpolates $f(x)$ at the $n+1$ distinct points $x_0, ..., x_n \in [a,b]$. Then $\forall\ x \in [a,b]$, $\exists\ \xi \in [a,b]$ such that

$$E(x) = f(x) - p(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi) \prod_{j=0}^{n} (x - x_j)$$

where ξ is an intermediate point between x_0 and x_n (Caporale and Cerrato, 2008).

If we are free to choose the interpolating points $x_0, ..., x_n$ within this interval, then the product $\prod\limits_{j=0}^{n}(x - x_j)$ can be minimized which in turn would minimize the interpolating error $E(x)$. This can be achieved by choosing interval as $[-1,1]$ and the interpolating points x_j as the Chebyshev points (Yang et. al , 2005). The following theorem gives an estimate of the error for the above case.

Theorem 2: Assume that $p(x)$ interpolates $f(x)$ at $x_0, x_1, ..., x_n$. Also assume that these $n+1$ interpolation points are the $(n+1)$ roots of the Chebyshev polynomial of degree $T_{n+1}(x)$, which are given by Eq.(8). Then $\forall\ x \in [-1,1]$,

$$|f(x) - p(x)| \leq \frac{1}{2^n (n+1)!} \max_{\xi \leq 1} |f^{n+1}(\xi)||$$

Our goal is not to approximate a function $p(x)$ on the interval $[-1,1]$, but to approximate the values of the function $p(x)$ corresponding to the discrete set of points given by Eq.(8). An arbitrary function $p(x)$ can be approximated in the interval $[-1,1]$ (Mason and Handscomb, 2002) by

$$p(x) = \sum_{k=0}^{n} c_k T_k(x), x \in [-1,1] \tag{14}$$

where the coefficients c_j are defined as

$$\begin{aligned} c_0 &= \frac{1}{n+1} \sum_{j=1}^{n+1} p(x_j) \\ c_k &= \frac{2}{n+1} \sum_{j=1}^{n+1} p(x_j) T_k(x_j), k = 1, ..., n \end{aligned} \tag{15}$$

5. Proposed Method

The signal encounters various types of artifacts during acquisition, transmission and storage. The noises introduced are due to power line interference (PLI), body movements, electrode contacts, electromagnetic field interference, and respiration movements etc (Acharya et. al, 2007). Presence of noises in ECG signals degrades the signal quality and thus affects the visual diagnosis and feature extraction. Thus, noise removal becomes an essential part in ECG preprocessing for better performance in ECG analysis and diagnosis.

Total variation denoising (TVD) is an approach for noise reduction and preservation of sharp edges of signals. The total variation (TV) of a signal measures how much the signal changes between signal values. Total variation denoising (TVD) is based on the principle that signals with excessive and possibly spurious detail have high total variation. According to this principle, reducing the total variation of the signal subject to it being a close match to the original signal, removes unwanted detail whilst preserving important details such as edges. Unlike a conventional low-pass filter, TV denoising is defined in terms of an optimization problem. Here we first apply the majorization-minorization approach to optimize the total variation in the ECG signals (Yadav and Ray, 2015).

The purpose of the segmentation is to divide a signal to several segments with the same statistical characteristics such as amplitude and frequency. A segmentation algorithm has a global perspective that it produces the best Piecewise Linear Representation (PLR) of data with the least amount of error (Keogh et. al, 2001). Since statistical characteristic of ECG changes with time, so ECG signals are considered as non-stationary signals. Analysis of stationary signal is easier as compared to non- stationary signal so signal segmentation is applied as a pre-processing step for non-stationary signal analysis. Hence, we apply the The Bottom Up algorithm, also called as iterative merge which begins by dividing the original time series data of length n into a large number of segments and is consequently merged into bigger segments until stopping criteria is met (Yadav and Ray, 2016).

Since we are processing one segment, our working domain is in the interval $[a, b]$. So, we first transform the interpolation interval $y \in [-1, 1]$ using

$$x = \frac{(b-a)y + (a+b)}{2} \tag{16}$$

This converts the interpolation problem for $f(x)$ on $[a, b]$ into interpolation problem for $f(x) = g(x(y))$ in $y \in [-1, 1]$. The Chebyshev points in the interval $y \in [-1, 1]$ are the roots of the Chebyshev polynomial $T_n(y)$, i.e.,

$$y_j = \cos\left(\frac{2j+1}{2n+1}\pi\right),\ 0 \le j \le n$$

The corresponding $n+1$ interpolation points in the interval $[a, b]$ using Eq.(16) are now

$$x_j = \frac{(b-a)y_j + (a+b)}{2},\ 0 \le j \le n \tag{17}$$

The interpolation error now is given by

$$|f(x) - p(x)| \le \frac{1}{2^n(n+1)!}\left|\frac{b-a}{2}\right|^{n+1} \max_{\xi\in[a,b]}\left|f^{(n+1)}(\xi)\right|$$

In our method we need to construct the function $f(x)$ using Eq.(13) with all the N ECG samples of one segment. Then we find the Chebyshev nodes and subsequently the interpolating polynomial using these nodes. Next, we calculate the error and if the error is not within our tolerance, we increase the order. Since an ECG signal sampled value may not be available at the Chebyshev nodes, we derive this value by linear interpolation using adjacent ECG sampled values. We continue doing these operations till our error criterion is met.

We apply the same technique to all the segments and model each of them independently using Chebyshev interpolation method. We present here an algorithm to show the steps of our method.

Algorithm Chebyshev_poly_approx=Chebyshev_poly_approx (N, ε, $f(x)$, $p(x)$, $[a, b]$)

Inputs: $p(x)$, $[a, b]$, $\varepsilon = 10^{-3}$

Outputs: $f(x)$

BEGIN algorithm

1. Fix the order n of the Chebyshev approximation.
2. Transform the Chebyshev nodes on the domain $[a, b]$ and find the zeros or the Chebyshev nodes x_j using Eq.(17)
3. Find the function value $f(x_j)$ by linear interpolation using the adjacent integral points around x_j
4. Construct interpolating polynomial $f(x)$ using Eq.(13).
5. Calculate error $E(x) = \max |f(x) - p(x)|$
6. If $E > \varepsilon$ then $n = n + 1$ and go to step 2

   END algorithm

6. Implementation and Results

An ECG signal is not linear, rather more curvaceous consisting of waves of various shapes. For testing the performance of our algorithm we conducted our tests in MATLAB environment. An ECG signal of duration 10 seconds with 8274 samples is taken from MIT-BIH (Goldberger et. al, 2000) arrhythmia database. Each file is sampled at 720Hz sampling frequency with 11 bits per sample of resolution. The denoised signal is obtained using the TVD approach. Since the ECG signal is quasi-stationary, segmentation plays very important role. The segmented points must be related with the diagonastical parameters, because they determine the diagonastical intervals and the wave amplitudes of the ECG. Peter Kovacs (Kovacs, 2012) had divided the ECG signal into 12 segments. In order to keep the model as simple as possible, number of segments should be minimal. So the number of segments has to be intelligently decided. Using Bottom Up approach we divide the denoised ECG signal into 10 segments so that the significant points and waves remain preserved at the time of approximation.

Since the shape of ECG is variable within and across patients, so all the segments cannot be approximated by a signal polynomial. Instead a number of polynomials are to be reconstructed depending upon the shape of segments. The proposed algorithm is implemented and tested over each segment of the ECG signal by choosing the order of polynomial in such a way so as to reduce the $MaxError$. Figure 2 to Figure 4 show the 10 original ECG signal segments and their approximated signals with their Chebyshev nodes marked as '$*$'. The original signal is shown in 'red' and the reconstructed signal is shown in 'blue'. Figure 5 shows the complete original and reconstructed signal.

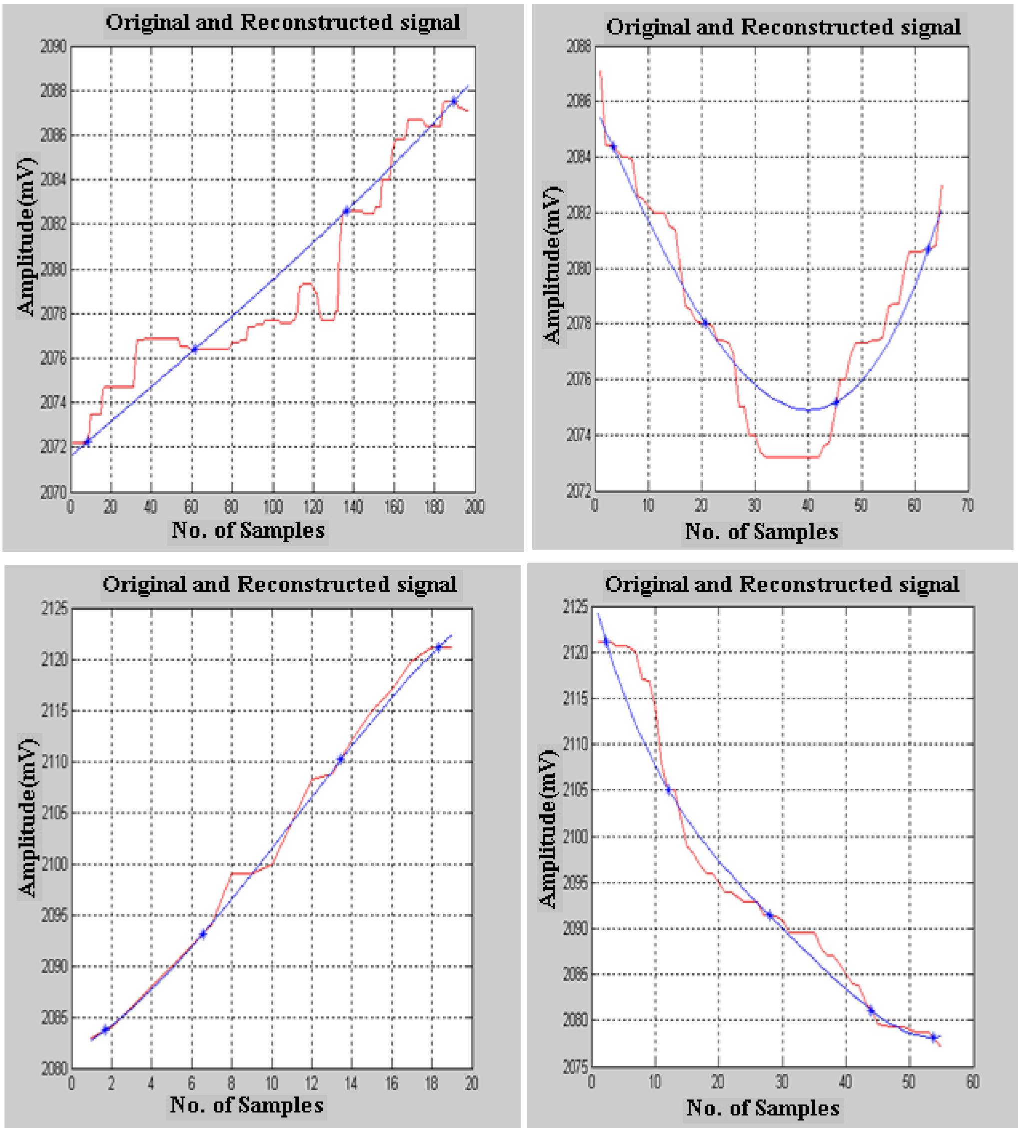

Figure 2. Segments 1 to 4 of Original and Reconstructed signals.

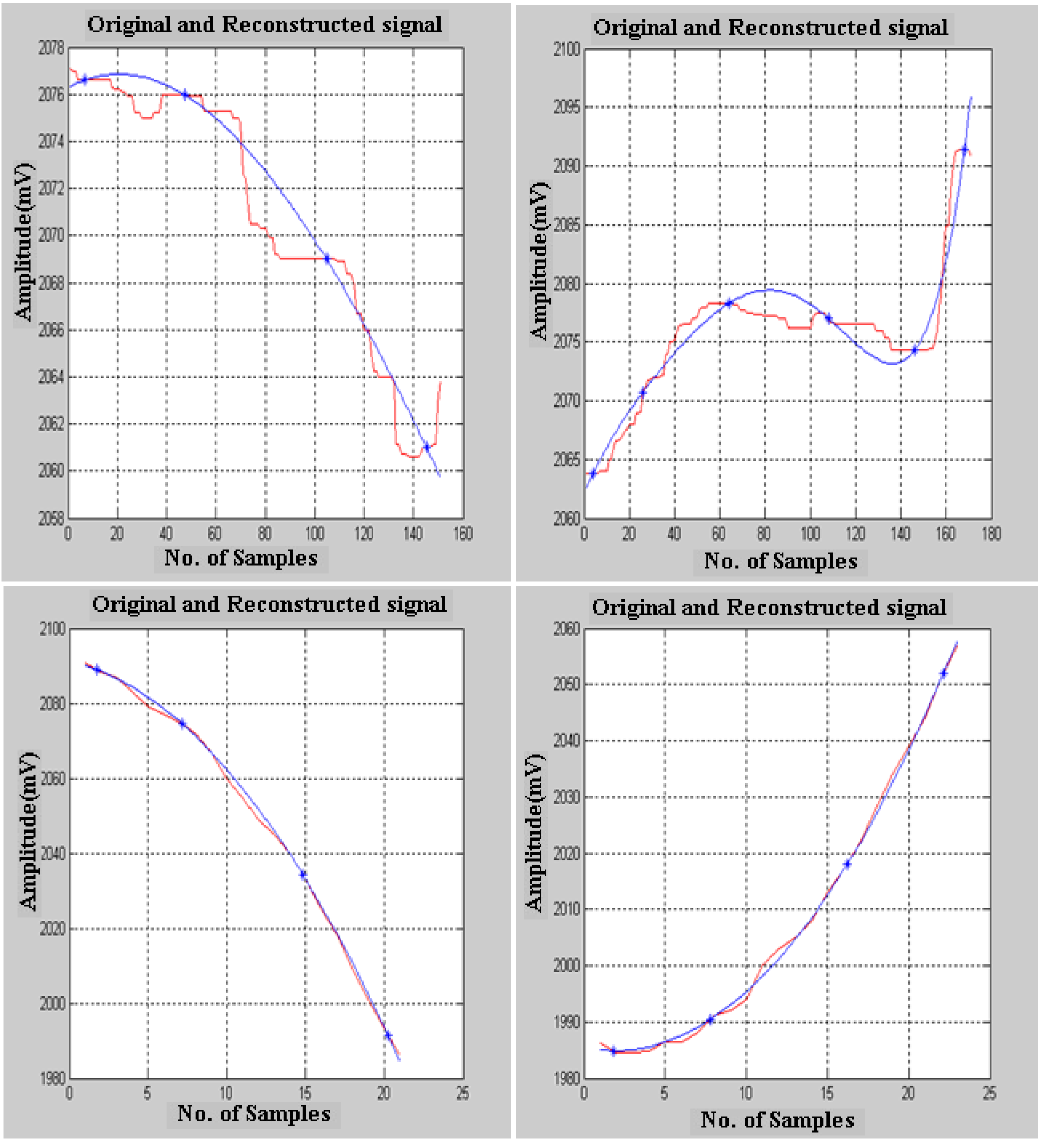

Figure 3. Segments 5 to 8 of Original and Reconstructed signals.

The performance of the Chebyshev approximation technique is measured in terms of CR, $RMSE$, $MaxError$, PRD and $PRDN$. Table I shows the results obtained for individual segments. The orders of the polynomials for each segment are chosen to retain the original shapes of the reconstructed signals. Compression ratios for linear sections are higher and can be approximated by third order Chebyshev polynomial with PRD approximately equal to 0.05. $MaxError$ for individual segments were also calculated and the highest is nearly 5.2 for segment number 6. ECG compression techniques with less than 10 % approximation error are considered to be medically

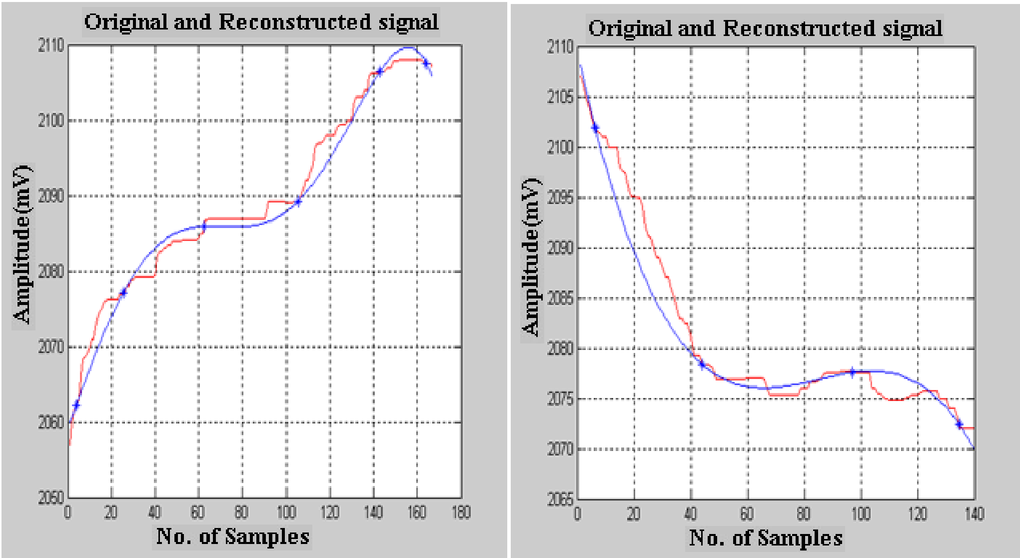

Figure 4. Segments 9 and 10 of Original and Reconstructed signals.

Table I. Performance metrics for all the segments.

Segment Number	Order n	N	CR	$RMSE$	$MaxError$	PRD	$PRDN$
1	3	197	65.67	1.5526	4.3348	0.0747	34.6824
2	3	65	21.67	1.2036	2.2892	0.0579	32.2823
3	3	19	06.33	0.9568	1.4826	0.0455	7.4977
4	4	55	14.75	2.7307	3.0925	0.1304	19.7871
5	3	151	50.33	1.3628	3.0040	0.0658	25.5228
6	5	171	34.20	1.7778	5.2056	0.0857	31.4943
7	3	21	7.00	1.4089	2.9302	0.0687	4.3420
8	3	23	7.67	0.9004	1.3611	0.0448	3.9164
9	5	167	33.40	1.8528	3.7423	0.0887	15.2980
10	3	140	46.67	2.3227	2.7732	0.1116	25.3066
Average	-	-	28.6683	1.6069	4.2298	0.0774	20.0130

accepted (Sandryhaila et. al, 2012). The maximum value of PRD is 0.13 for segment number 4 and average PRD obtained is 0.0774 which is also acceptable for ECG compression.

Similar approximation technique was used in (Yadav and Ray, 2013), but it was used on a simulated standard ECG signal with absolutely no noise, where one cycle of ECG signal was

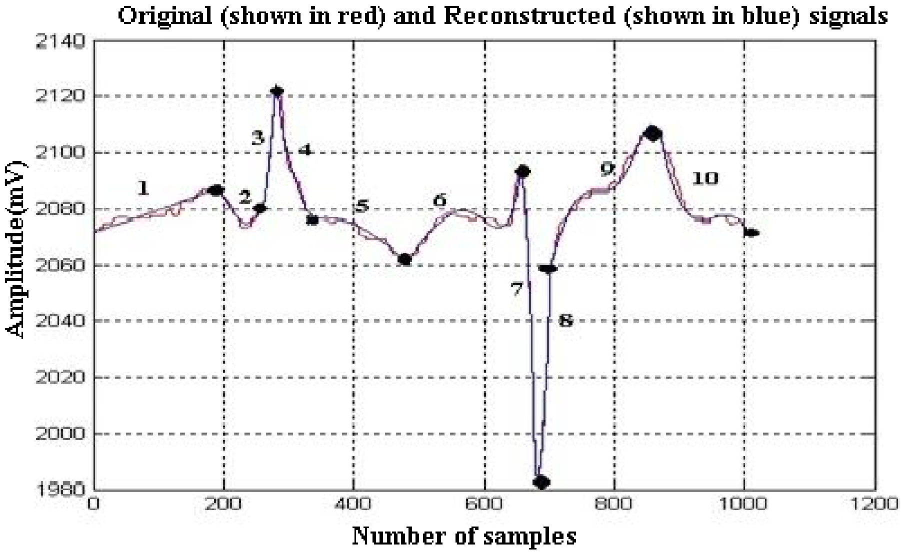

Figure 5. Original and Reconstructed ECG Signal with all its 10 segments.

segmented into seven sections on the basis of shape and duration of various waves of ECG. Hence, we have kept it outside the purview of our comparison.

The values of performance metrics obtained by the proposed method are much less than those shown in (Fira and Goras, 2008) with different ECG compression techniques, viz., Wavelet and Huffman, JPEG2000, SPHIT and other traditional techniques for one cycle of ECG signal. The mean value of CR obtained in (Fira and Goras, 2008) is 18.27 which is inferior than the average CR of 28.67 obtained by the proposed method. In (Tchiotsop et. al, 2007) ECG signals were approximated using Jacobi polynomials, where the highest compression achieved was 11. Hermite functions (Sandryhaila et. al, 2012) were also used to compress QRS complex of ECG signals and the average CR achieved was 11 with 25% approximation error. The results obtained by Sandryhailla et. al. were better than the results obtained by other transformation techniques. In (Jokic et. al, 2011) polynomial models of ECG signals were developed where the lowest and highest PRD observed were 3.5 and 10.8 respectively. In (Zahhad et. al, 2010) Discrete Wavelet Transform was used to compress ECG signals where the highest CR of 40 was achieved with PRD of 2.7, and lowest CR was 3.4 with PRD of 0.2 for one dataset. Hence, we can claim the proposed technique is quite suitable for ECG compression.

7. Conclusion

In this paper, a model is designed to compress ECG signals using Chebyshev polynomials. The signal was taken from MIT-BIH database which was denoised using the TVD-MM approach. Bottom-Up technique was then used to find the break points. The individual segments obtained were then approximated using Chebyshev polynomials. From the results, it was observed that the order of

Chebyshev polynomials depends upon the shape of various sections of ECG wave. Waves having zero or constant slope can be approximated by lower order Chebyshev polynomials, whereas waves having variable slopes require higher order Chebyshev polynomials. The approximated models are evaluated in terms of CR, $RMSE$, $MaxError$, PRD and $PRDN$. It was observed that the results obtained were superior than those reported in the existing literature. Also accuracy can be increased by breaking the complete signal into more number of segments at the cost of CR.

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